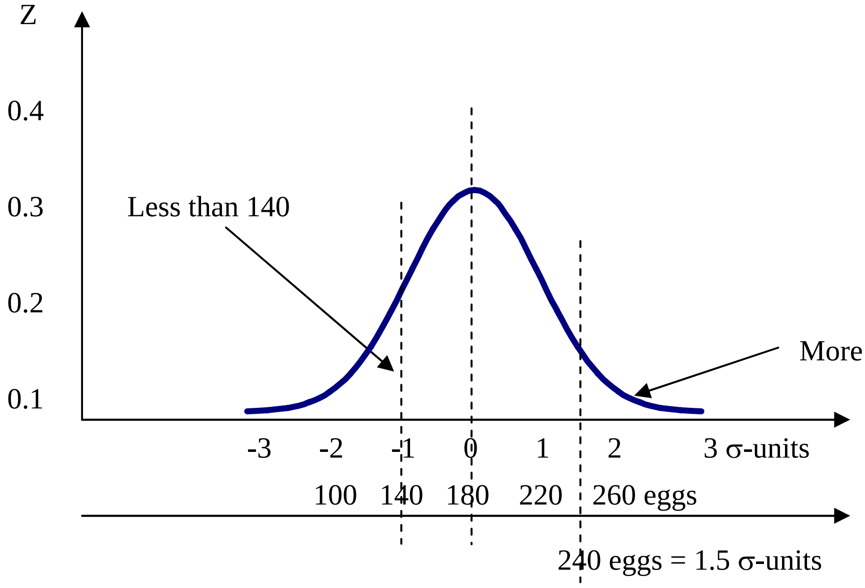
Selection and genetic gain - solutions to exercises

1. a) Construct a curve (according to the table):

x_0	z
2.88	0.0063
0.995	0.243
0.00	0.399
-2.054	0.048



b) $x_0 = \frac{240-180}{40} = 1.5\sigma - units$

Interpolation in table:

$$\begin{array}{ccc} \underline{x_0} & & \underline{v} \\ \left.\begin{array}{l}1.555\\1.500\\1.476\end{array}\right\} & 0.024 \Big\} 0.079 & \left.\begin{array}{l}6\\v\\7\end{array}\right\} \quad \Delta v \Big\} \quad -1 \end{array}$$

$$\frac{\Delta v}{-1} = \frac{0.024}{0.079}$$

$$\Delta v = -0.304$$

$$v = 7 + \Delta v = 7 - 0.304 = 6.7$$

$x_0 = 1.5 \rightarrow v = 6.7\%$

6.7% of the hens produce more than 240 eggs.

c) $x_0 = \dfrac{140-180}{40} = -1\sigma - unit$

Interpolation in the table gives x_0 = -1 ⟶ v = 84.1% of the hens are kept for breeding purposes

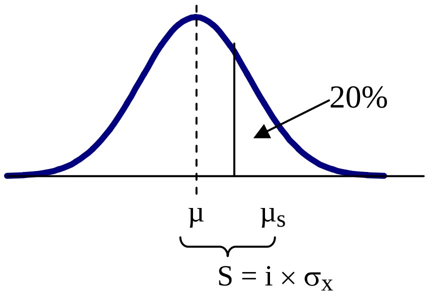
x_0 = -1 gives z = 0.242

Selection intensity: $= \dfrac{0.242}{0.841} = 0.29$ (see also table)

Selection difference: $S = I \times \sigma_x = 0.29 \times 40 = 11.6$ eggs

The mean for the selected animals exceeds the population mean with 11.6 eggs (μ_s = 11.6 + 180 = 191.6 eggs).

2. a) $v = \frac{2000}{10000} = 20\% \rightarrow i = 1.4$



$S = i \times \sigma_X = 1.4 \times 1000 = 1400$

$\mu_S = \mu + S$

$5500 = \mu + 1400 \qquad \mu = 4100$

Average milk yield for the whole population = 4100 kg

b) $40\% \rightarrow I = 0.966$

$S = i \times \sigma_X = 0.966 \times 1000 = 966$

$\mu_S = 4100 + 966 = 5066$ kg

Average yield for the 4000 best cows = 5066 kg

c) $v = 40\% \longrightarrow x_0 = 0.253$ σ-units

The limit is at $4100 + (0.253 \times 1000) = 4353$ kg

3. Population size: $100 \times 1.8 \times 0.5 = 90$ ewe lambs

Number of selected: $0.20 \times 100 = 20$ ewe lambs

Percentage selected: $\frac{20}{90} \times 100 = 22.22\%$

Interpolation in the table gives I = 1.340

Correction for populations with less than 500 individuals.

$$i' = i - \frac{0.25}{number\ of\ selected} = 1.340 - \frac{0.25}{20} = 1.328$$

$$h^2 = \frac{\sigma_A^2}{\sigma_P^2}; \quad \sigma_P = \sqrt{\frac{\sigma_A^2}{h^2}} = \sqrt{\frac{4.8}{0.3}} = \sqrt{16} = 4$$

Selection differential $S = i' \times \sigma_P = 1.328 \times 4 = 5.312$

4. v = 0.60 gives I = 0.644

σ_P = 500 μ = 5500 h_2 = 0.25 L = 4.5 years

a) S = i × σ_P = 0.644 × 500 = 322 kg

μ_S = μ + S = 5500 + 322 = 5822 kg

b) $$\Delta T_{year} = \frac{r_{TI} \times i \times \sigma_T}{L} = \frac{r_{TI} \times i \times \sqrt{h^2} \times \sigma_P}{L} = \frac{r_{TI} \times \sqrt{h^2} \times S}{L}$$

For performance testing with one observation: $r_{TI} = \sqrt{h^2}$

$$\Delta T_{year} = \frac{h^2 \times S}{L} = \frac{0.25 \times 322}{4.5} = 17.9\ kg / year$$

5. a) v = 50% gives i = 0.798

$$h^2 = \frac{\sigma_A^2}{\sigma_P^2} = \frac{\sigma^2}{12^2} = 0.25$$

$\Delta T = h^2 \times i \times \sigma_P = 0.25 \times 0.798 \times 12 = 2.394$ units/generation

b) $$\Delta T_{year} = \frac{1 \times 105}{100} = \frac{0.25 \times i \times 12}{4}\ units / year$$

$$i = \frac{4 \times 105 \times 1}{100 \times 0.25 \times 12} = 1.4$$ which corresponds to 20% selected

6. a) Performance testing:

$b = h^2 = 0.4$ $\qquad r_{TI} = \sqrt{0.4} = 0.63$

2000 animals can be tested – 100 are selected

$$v = \frac{100}{2000} = 5\% \qquad i = 2.063$$

b) Progeny testing:

$$\frac{1 + (10-1)\,0.25 \times 0.4}{10} \times b = 0.5 \times 0.4$$

$$b = \frac{10 \times 0.5 \times 0.4}{1 + 9 \times 0.25 \times 0.4} = 1.0526$$

$$r_{TI} = \sqrt{0.5 \times 1.0526} = 0.73$$

2000 animals – 10 progenies/sire ⟶ 200 animals can be tested

$$v = \frac{100}{200} = 50\% \qquad i = 0.798$$

c) $$h = \sqrt{0.4} = \frac{\sigma_T}{\sigma_P} = \frac{\sigma_T}{10}; \quad \sigma_T = 10 \times \sqrt{0.4} = 6.3kg$$

$\Delta T = r_{TI} \times i \times \sigma_T$ per generation

Performance testing: $\Delta T = 0.63 \times 2.063 \times 6.3 = 8.2$ kg

Progeny testing: $\Delta T = 0.73 \times 0.798 \times 6.3 = 3.7$ kg

c) Progeny testing will increase the generation interval (L). This means that performance testing will be comparatively even more favourable when the annual genetic change is considered. It can however be discussed if $r_{TI} = 0.63$ (which we get here for performance testing) is enough.

7. a) $I_1 = b_1X_1$ $\sigma_{T_2} = \sqrt{h_2^2 \times \sigma_{P_2}^2} = 0.28$

$T = A_2$

$\Delta T_2 \mid I_1 = r_g \sqrt{h_1^2} \times i \times \sigma_{A_2} = -0.85\sqrt{0.35} \times 1.4 \times 0.28 =$

$= -0.197 \simeq -0.2$ MJ/kg weight gain

b) $\dfrac{\Delta T_2 \mid I_1}{\Delta T_2 \mid I_2} = \sqrt{(r_g)^2 \times \dfrac{h_1^2}{h_2^2}} = \sqrt{(-0.85)^2 \times \dfrac{0.35}{0.40}} = 0.795 \rightarrow 80\%$

8. σ_A^2 is derived from the relation $h^2 = \dfrac{\sigma_A^2}{\sigma_P^2}$

$\sigma_A = \sqrt{h^2 \times \sigma_P^2}$

$\sigma_{A_1} = \sqrt{0.25 \times 738^2} = 369$

$\sigma_{A_2} = \sqrt{0.25 \times 31.0^2} = 15.50$

$\sigma_{A_3} = \sqrt{0.25 \times 24.5^2} = 12.25$

Selection of the 50% best cows gives selection intensity i = 0.798 (see Table).

The direct selection effect on trait 1 is for individual selection on I_1:

$\Delta T_1 \mid I_1 = \sqrt{h_1^2} \times i \times \sigma_{A_1}$

The correlated selection effect for trait 2 is then:

$\Delta T_2 \mid I_1 = r_{g_{12}} \times \sqrt{h_1^2} \times i \times \sigma_{A_2}$

a) Direct selection for milk yield (X_1) gives

$$\Delta T_1 \,|\, I_1 \quad = \quad 0.5 \times 0.798 \times 369 \quad = \quad 147.23$$

Selection for only milk yield gives a correlated effect in fat yield:

$$\Delta T_2 \,|\, I_1 \quad = \quad 0.95 \times 0.5 \times 0.798 \times 15.50 \quad = \quad 5.88$$

Correlated effect in protein yield:

$$\Delta T_3 \,|\, I_1 \quad = \quad 0.90 \times 0.5 \times 0.798 \times 12.25 \quad = \quad 4.40$$

b) Direct selection for fat yield (X_2) gives:

$$\Delta T_2 \,|\, I_2 \quad = \quad 0.5 \times 0.798 \times 15.50 \quad = \quad 6.18$$

Correlated effect in milk yield:

$$\Delta T_1 \,|\, I_2 \quad = \quad 0.95 \times 0.5 \times 0.789 \times 369 \quad = \quad 139.87$$

Correlated effect in protein yield:

$$\Delta T_3 \,|\, I_2 \quad = \quad 0.82 \times 0.5 \times 0.789 \times 12.25 \quad = \quad 4.01$$

c) Direct selection for protein yield gives:

$$\Delta T_3 \,|\, I_3 \quad = \quad 0.5 \times 0.789 \times 12.25 \quad = \quad 4.89$$

Correlated effect in milk yield:

$$\Delta T_1 \,|\, I_3 \quad = \quad 0.90 \times 0.5 \times 0.789 \times 369 \quad = \quad 132.51$$

Correlated effect in fat yield:

$$\Delta T_2 \,|\, I_3 \quad = \quad 0.82 \times 0.5 \times 0.789 \times 15.50 \quad = \quad 5.07$$

9. The selection intensity (i) increases but the accuracy (r_{TI}) decreases, unless the heritability is very high. The generation interval (L) decreases. Genetic gain increases if the gain in *i* and *L* is greater than the loss of accuracy.

10. v_S = 1% From the table: i_S = 2.665

v_D = 50% i_D = 0.798

$$\Delta T \quad = \quad \frac{R_S + R_D}{2}$$

$$R_S \quad = \quad r_{TI_S} \times i_S \times \sigma_T \quad = \quad 0.96 \times 2.665 \times 1000 \quad = \quad 2.558 \textit{ pounds}$$

$$R_D \quad = \quad r_{TI_D} \times i_S \times \sigma_T \quad = \quad 0.69 \times 0.798 \times 1000 \quad = \quad 551 \textit{ pounds}$$

$$\Delta T \quad = \quad \frac{2.558 + 551}{2} \quad = \quad 1555 \textit{ pounds}$$

11.

	Ewes **D**	**Rams** **S**
% selected	20	2
i	1.400	2.421
σ_p, kg	4	4
$i \cdot \sigma_p$ = S, kg	5.6	9.684
$h^2 \cdot$ S = R, kg	1.68	2.905
L, year	4	1.5

$$\Delta T_y = \frac{R_D + R_S}{L_D + L_S} = \frac{1.68 + 2.905}{4 + 1.5} = 0.834\ kg\,/\,year$$

12. $\sigma_T = \sigma_A = \sqrt{h^2} \times \sigma_P$

Path in selection	$\mathbf{r_{TI}}$	**i**	$\boldsymbol{\sigma_T}$	**R = $\mathbf{r_{TI} \cdot i \cdot \sigma_T}$**	**L**
F-S	0.877	2.063	400	723.7	8
F-D	0.877	1.400	400	491.1	6
M-S	0.598	1.755	400	419.8	7
M-D	0.500	0.350	400	70.0	7
Total				1704.6	28

$$\Delta_{T_y} = \frac{\Sigma R}{\Sigma L} = \frac{1704.3}{28} = 60.88\ kg\,/\,year$$

$$\Delta_{T_y} = \frac{60.88}{5000} \times 100 = 1.2\%\ per\ year$$